Optimal Autostabilizer for a Supersonic Fighter Aircraft

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Abstract

A MODERN approach to supersonic fighter aircraft design is the control configured fighter (CCF).[1] In this approach, an unstable airframe is purposely designed with a view to obtaining superior handling qualities, and an autostabilizer is incorporated as an integral part of the design to obtain stability. Autostabilizers have hitherto been designed using classical control system design techniques based on Bode plots, root locus plots, etc. In this investigation, the modern optimal control theory has been employed to design an autostabilizer for a supersonic fighter aircraft. A novel method based on Krotov's method of approaching the solution of Bellman's equation of dynamic programming has been developed to solve the optimal control problem. The resulting suboptimal bang-bang control law is remarkable inasmuch as 1) it contains only a linear combination of the state variables and, consequently, is easy to implement and 2) the results obtained with it are excellent.[2]

Contents

Application of optimal control theory to autostabilizer design involves four steps as described in the following.

Derivation of State Equations for the Aircraft

Linearized equations describing the longitudinal short-period mode of the aircraft in question have been derived using the standard procedure.[3] These equations may be expressed in the standard form

$$\dot{x} = Ax + bu \tag{1}$$

where

$$A = \begin{bmatrix} -0.84 & 0 & 0.995 & -0.107 \\ 0 & 0 & 1 & 0 \\ -56.9 & 0 & -0.5 & -44.2 \\ 0 & 0 & 0 & -20 \end{bmatrix}$$

and

$$b = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 20 \end{bmatrix}$$

The state variables chosen are: x_1 = angle of attack α, x_2 = pitch angle θ, x_3 = pitch rate $\dot{\theta}$, and x_4 = elevator deflection δ_e. The electrical input e_{δ_e} to the elevator system has been chosen as the control variable u as "fly-by-wire" control is envisaged. The elevator system dynamics have thus been included in aircraft dynamics. This explains why our model of the aircraft is a fourth-order model rather than the third-order model usually assumed.

Selection of Performance Index

The performance index (PI) selected in our investigation

$$J = x_1^2(t_f) + x_2^2(t_f) + x_3^2(t_f) + \int_0^{t_f} \left(x_1^2(t) + x_2^2(t) \right) \mathrm{d}t \tag{2}$$

subject to the constraint $|u(t)| \le U_{max.}$ since the control objective is to correct deviations caused by disturbances as early as possible with minimum final deviation.

Derivation of the Suboptimal Control Law

It needs to be emphasized that Eq. (2) is *not* the usual quadratic performance index inasmuch as it does not contain

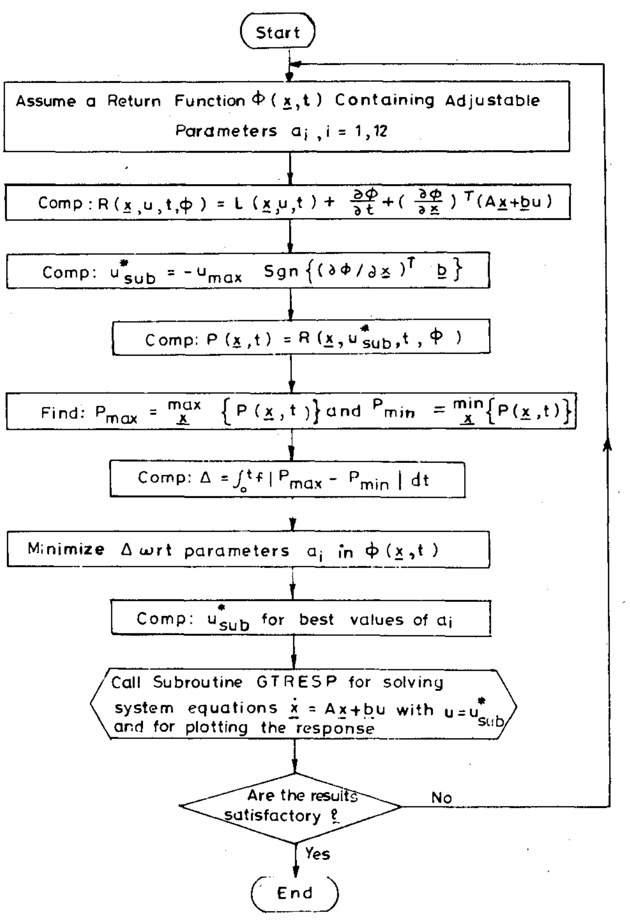

Fig. 1 Novel algorithm for obtaining suboptimal control law.

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Table 1 Improvement in PI

Initial conditions				P.I. with autostabilizer	P.I. without autostabilizer	Improvement ratio
$\alpha(0)$	$\theta(0)$	$\dot{\theta}(0)$	$\delta_e(0)$			
0.1	0.1	1.0	0.0	0.008924	0.110139	1:12.37
−0.1	−0.1	1.0	0.0	0.058062	0.831403	1:14.3

Table 2 Comparison of three investigations on optimal autostabilizer design

Series no.	Item	Moranker's investigation	Moharil's investigation	Present investigation
1	Model	3rd order	4th order	4th order
2	Limits on elevator deflection	+0.288 rad −0.131 rad	+0.2 rad −0.2 rad	+0.1 rad −0.1 rad
3	PI	$\int_0^{t_f} dt$	$\int_0^{\infty} (x^TQx+u^TRu)dt$	$F[x(t_f)] + \int_0^{t_f} (x^TQx)dt$
4	Control law	Suboptimal, bang-bang type, closed loop	Optimal, linear, closed loop and suboptimal, dual mode, closed loop	Suboptimal, bang-bang type, closed loop
5	Final time (t_f)	–	∞	0.5 s
6	$T_{½}$	0.285 s	0.385 s	0.33 s

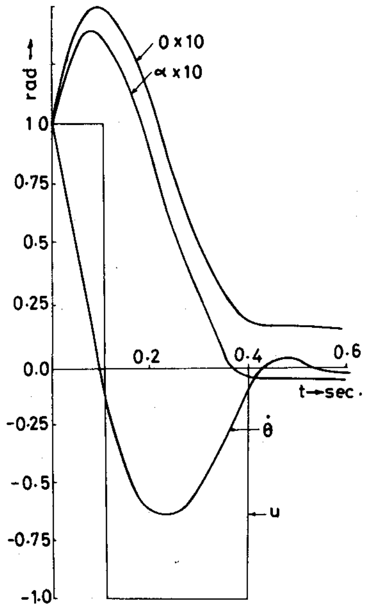

Fig. 2 Transient response for the best suboptimal control law designed.

a quadratic term in the control variable $u(t)$. Naturally, the resulting optimal control law will *not* be the usual linear feedback law; in fact, it will be of the *bang-bang* type. Since initial conditions change as disturbances occur from time to time, we must have a feedback control law. To this end, we have developed a novel algorithm based on Krotov's method[4] for solving Bellman's equation of dynamic programming. The algorithm is explained in the flow chart of Fig. 1.

Results Obtained

Our novel algorithm was implemented on three trial return functions $\Phi(x,t)$ using a combination of random search and conjugate gradient methods. The best suboptimal control law obtained was:

$$u^*_{\text{sub}} = -0.1\text{sgn}[-0.7634\,x_1 + 0.4465\,x_2 - 0.18\,x_3 + 0.4176\,x_4] \tag{3}$$

The time response corresponding to this control law is shown in Fig. 2. It is seen that the response in the control interval is very good.

The goodness of this suboptimal control law has been tested in two stages: 1) to compare the performance of the aircraft without and with control (the results are shown in Table 1) and 2) to compare the results with two similar investigations carried out by Morankar[5] and Moharil[6] (the results are shown in Table 2). It is seen that the results of test 1 are excellent, whereas the results of test 2 are very favorable.

References

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